LETTERS TO THE EDITOR

Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the twentieth of the preceding month; for the second issue, the fifth of the month. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Communications should not in general exceed 600 words in length.

Note on Neutron-Proton Exchange Interaction

The matrix elements of the interaction between a proton with coordinates $\mathbf{x}_1$ and a neutron with coordinates $\mathbf{x}_2$ as proposed by Majorana[1] may be written as

$$(\mathbf{x}_1\mathbf{x}_2|\,V|\,\mathbf{x}_1'\mathbf{x}_2')=-J(|\mathbf{x}_1-\mathbf{x}_2|)\delta(\mathbf{x}_1-\mathbf{x}_2')\delta(\mathbf{x}_2-\mathbf{x}_1'). \quad (1)$$

This interaction assumption corresponds to an interchange of the positional coordinates $(\mathbf{x}_1, \mathbf{x}_2)$ of the two particles; clearly this interchange will entail a shift in the center of mass if the mass of the proton, m_1, is not the same as the mass of the neutron, m_2. A similar effect on the center of mass is also characteristic of the Heisenberg interaction assumption[2] in which both spin and positional coordinates are interchanged, and of any "mixed" Heisenberg-Majorana interaction. While the difference between the masses of the proton and neutron according to present values is small, the difficulty introduced by the shift in the center of mass has perhaps a theoretical interest and has been discussed by Breit and Wigner.[3]

This center of mass difficulty is made at once evident by considering the matrix elements of the exchange interaction in terms of momentum variables. Thus, the transformation of (1) gives

$$(\mathbf{p}_1\mathbf{p}_2|\,V|\,\mathbf{p}_1'\mathbf{p}_2')=f(|\mathbf{p}_2'-\mathbf{p}_1|)\delta(\mathbf{p}_1'-\mathbf{p}_1+\mathbf{p}_2'-\mathbf{p}_2), \quad (2)$$

in which $\mathbf{p}_1$ is a momentum of the proton, and $\mathbf{p}_2$ a neutron momentum and the form of f depends on the particular form assumed for J in (1). If the matrix elements (2) are now written as functions of the total momentum $\mathbf{P}=\mathbf{p}_1+\mathbf{p}_2$ and the relative momentum $\mathbf{p}=\mu(\mathbf{p}_2/m_2-\mathbf{p}_1/m_1)$, where $\mu=m_1m_2/(m_1+m_2)$, (2) becomes

$$(\mathbf{pP}|\,V|\,\mathbf{p}'\mathbf{P}')=f(|\mathbf{p}+\mathbf{p}'+\mathbf{P}(m_2-m_1)/(m_1+m_2)|)\delta(\mathbf{P}-\mathbf{P}'), \quad (3)$$

in which form the flaw in the exchange interaction assumption is obvious: the matrix elements of the interaction are explicit functions of the momentum $\mathbf{P}$ of the center of mass so that it is not possible to separate the energy or the wave function into parts characteristic, respectively, of the internal motion and the motion as a whole. (3) is to be regarded as a consequent interaction law only insofar as one may put $m_1=m_2$, for which case

$$(\mathbf{pP}|\,V|\,\mathbf{p}'\mathbf{P}')=f(|\mathbf{p}+\mathbf{p}'|)\delta(\mathbf{P}-\mathbf{P}'), \quad (3')$$

and the difficulty mentioned above disappears.

There are many possible ways of modifying the interaction assumption (1) or (2) without losing the essential feature of the exchange, and without introducing at the same time any error due to a difference in masses. One simple possibility is to take instead of (2) the following

$$(\mathbf{p}_1\mathbf{p}_2|\,V|\,\mathbf{p}_1'\mathbf{p}_2')=f\left(\frac{\mu}{m_1m_2}|m_1(\mathbf{p}_2'+\mathbf{p}_2)-m_2(\mathbf{p}_1'+\mathbf{p}_1)|\right)\delta(\mathbf{p}_1'-\mathbf{p}_1+\mathbf{p}_2'-\mathbf{p}_2). \quad (4)$$

Then,

$$(\mathbf{pP}|\,V|\,\mathbf{p}'\mathbf{P}')=f(|\mathbf{p}+\mathbf{p}'|)\delta(\mathbf{P}-\mathbf{P}'). \quad (5)$$

The identity of (5) and (3′) proves the interesting result that the wave functions $\psi(p)$ and the energy values $E(P)$ found by using the Majorana interaction under the assumption $m_1=m_2$ are exactly the same as the corresponding wave functions and energy values deduced from the modified interaction which is valid regardless of the values of m_1 and m_2.

The same conclusions as given above should of course follow from consideration of the interaction in a coordinate representation. In fact, the coordinate representation of (4) is

$$(\mathbf{x}_1\mathbf{x}_2|\,V|\,\mathbf{x}_1'\mathbf{x}_2')=-J(|\mathbf{x}_1-\mathbf{x}_2|)\times\delta\left[\mathbf{x}_1-\mathbf{x}_2'+\frac{m_1-m_2}{m_1+m_2}(\mathbf{x}_1-\mathbf{x}_2)\right]\times\delta\left[\mathbf{x}_2-\mathbf{x}_1'+\frac{m_1-m_2}{m_1+m_2}(\mathbf{x}_1-\mathbf{x}_2)\right], \quad (6)$$

which is the same interaction as has been proposed by Breit and Wigner. If the wave function for a neutron and a proton is written as $\psi=u(\mathbf{X})g(\boldsymbol{\xi})$ where $\mathbf{X}=(m_1\mathbf{x}_1+m_2\mathbf{x}_2)/(m_1+m_2)$ and $\boldsymbol{\xi}=\mathbf{x}_1-\mathbf{x}_2$, and if V is the operator defined by (6), $V\cdot u(\mathbf{X}')g(\boldsymbol{\xi}')=-J(|\boldsymbol{\xi}|)u(\mathbf{X})g(-\boldsymbol{\xi})$, while the operator defined by (1) is such that $V\cdot u(\mathbf{X}')g(\boldsymbol{\xi}')=-J(|\boldsymbol{\xi}|)u(\mathbf{X}+(m_2-m_1)\boldsymbol{\xi}/(m_2+m_1))g(-\boldsymbol{\xi})$.

It is clear that it is possible to extend the exchange interaction law so as to include the case of two particles of different masses without adding any essential complication. The greater complexity of (6) as compared with (1) is only apparent and should be regarded as characteristic of the particular coordinate system used. Although there may be other reasons for supposing that the exchange interaction is accurate only insofar as $(m_1-m_2)/(m_1+m_2)$ can be neglected in comparison with unity, it does not appear that such a limitation[3] can be found by examining the exchange hypothesis itself for consistency and simplicity.

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March 11, 1936.

[1] E. Majorana, Zeits. f. Physik **82**, 137 (1933).
[2] W. Heisenberg, Zeits. f. Physik **77**, 1 (1932); **78**, 150 (1932).
[3] G. Breit and E. Wigner, Phys. Rev. **48**, 918 (1935).